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The $K\pi$ and $\pi\pi$ S-wave amplitude from D-meson decays

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Abstract. The issue of $K\pi$ and $\pi\pi$ S-wave amplitude is addressed using decays of D-mesons. Model-independent measurements of the phases of the $\pi^+\pi^+$ and $K^-\pi^+$ S-wave amplitude from $D^+ \to \pi^-\pi^+\pi^+$ and $D^+ \to K^-\pi^+\pi^+$ decays are discussed. The result indicates a deviation from the phase of the $K^-\pi^+$ S-wave amplitude obtained by scattering experiments. This could be interpreted as an indication of the presence of 3-body final-state interaction, or in other words, that the phases from production and scattering process cannot be directly compared.

PACS. 11.80.Et Partial-wave analysis – 11.80.La Multiple scattering – 13.30.Eg Hadronic decays – 13.30.Ce Leptonic, semileptonic, and radiative decays

1 Introduction

Hadronic decays of heavy flavor are a key tool for the studies of a number of phenomena, from hadron dynamics to CP violation. Specifically, decays of D-mesons to light hadrons is a unique source of information on the dynamics of the strong interaction at low energy.

In hadronic multi-body D decays resonances are abundantly produced. The bulk of the hadronic decay rate can be described by combining simple tree level quark diagrams and regular $q\bar{q}$ resonances. The memory of the quark content of the initial state is preserved: in the case intermediate channels having a vector, axial-vector or tensor states, there is a strong correlation between the final state quarks of the D decay and the quark content of the produced resonances. Decays of D-meson could, thus, be seen as an anti-glueball filter. Extending to the scalar sector the above-mentioned correlation, one could infer the quark content of the scalar states. For instance, the large fraction of the $\sigma\pi$ component in $D^+ \to \pi^-\pi^+\pi^+$ and the absence of such channel in the $D_s^+ \to \pi^-\pi^+\pi^+$ suggests a small $\bar{s}s$ and a strong $\bar{n}n$ component the σ wave function.

Another advantage of D decays is that one can continuously cover the $K\pi$ and $\pi\pi$ spectra from threshold up to 1.7 GeV ($M_D - m_\pi$), filling the gaps on the existing data on $\pi\pi$ and $K\pi$. Data from K_{e4} decays go from $\pi\pi$ threshold up to 380 MeV, while the CERN-Munich data starts only at 580 MeV. In the case of the $K\pi$, the LASS data starts only at 825 MeV.

An important feature of D decays is the absence of the Adler zeroes, which suppress the production of the lowest-mass scalar resonances σ and κ in $\pi\pi$ and $K\pi$ scattering. Moreover, the scalar component is very large in the hadronic decays of the D-meson to three-body final states with a pair of identical particles. These features make the $D^+, D_s^+ \to \pi^-\pi^+\pi^+$ and $D^+ \to K^-\pi^+\pi^+$ the golden modes for studies of the $\pi\pi$ and $K\pi$ S-wave amplitude.

2 The $\pi\pi$ S-wave amplitude

A few years ago the E791 Collaboration published a series of Dalitz plot analyzes showing evidence for low mass, broad scalar states, identified as the σ [1] and κ [2]. Soon after, CLEO [3], BaBaR [4], Belle [5], using D decays, and BES [6], using J/ψ decays, showed further evidence for these states in different reactions. Nowadays, the existence of these broad scalars is well established.

In all of the above analyses the S-wave component of the decay amplitude included a Breit-Wigner for the σ and for the κ. It is well known that the Breit-Wigner is not a correct representation for such states [7] but, nevertheless, it yields very good fits to the data in all cases. One should look at the Breit-Wigner as an effective representation of the amplitude for real values of energy, where we have the experimental data. Even though there is a remarkable agreement between the σ Breit-Wigner parameters obtained from different analyzes of D decays (BES results using J/ψ decays yield a somewhat different set of parameters), one should not use them to extract the σ pole position.

There has been an intense debate on how to represent the S-wave amplitude. The basic point here is that the phase of the S-wave amplitude from D decays does not

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match that of $\pi\pi$ scattering. One possible reason for this discrepancy would be the fact that the experimentalists adopted a wrong model in their analysis. There has been attempts to reconcile the S-wave phases. An example is the work done by J. Oller [8]. The argument is as follows. If one writes the amplitude as $N(s)/D(s)$, where $N(s)$ is a real function accounting for the production of the $\pi\pi$ pair and $D(s)$ a complex function describing the $\pi\pi$ final-state interaction, then different process like $\pi\pi$ scattering and D decays would have different $N(s)$, but should share the same function $D(s)$. According to this point of view $D(s)$ would be a universal function for all process involving a $\pi\pi$ pair. After all, if a resonance is a real particle, it should have the same mass and width in whichever process it appears. A good fit to E791 data is obtained in [8], but at the expense of very large interference terms. In practice, it is a bit difficult to distinguish between the non-resonant contribution and broad structures with no distinct angular distribution in the Dalitz plot. The magnitudes and phases of the non-resonant term, and the mass and width of broad states are correlated, so can have similar distributions constructed from different combinations of these parameters.

There has been also attempts to use the K-matrix approach, which is also based on the assumption of an isolated $\pi\pi$ system. An example is the analysis of Fermilab FOCUS experiment [9]. Again, a good fit is obtained, but the same data can be also fitted with a σ Breit-Wigner, yielding mass and width which is very similar to the E791 parameters. There are several problems with the K-matrix approach: the physical interpretation is a bit obscure, and there is enough freedom in the model for the $\pi\pi$ production to accommodate the σ peak without considering it explicitly as one of the K-matrix poles.

For an isolated $\pi\pi$ system, the matching of the phases would be a consequence of the Watson theorem [10]. In spite of being often used, this assumption is far from trivial, since the $\pi\pi$ is embedded in a multi-body, strongly interacting final state. Actually, there is no experimental evidence to support this assumption.

From the experimental side, a high statistics, model-independent measurement of the S-wave phase from D decays is in order. It would be very interesting also to have a comparative study of the semi-leptonic decays $D^+ \to \pi^-\pi^+l^+\nu$, where the $\pi\pi$ system is really isolated.

Recently, a model-independent measurement of the S-wave phase was performed on the E791 data by Miranda and Bediaga [11], based on the so-called amplitude difference method. This method uses the interference between the S- and the D-waves (essentially the σ and $f_2(1270)$) in the decay $D^+ \to \pi^-\pi^+\pi^+$ to extract the S-wave phase with no assumption. The result confirmed the resonant behavior at low $\pi\pi$ mass, with a phase motion that is different from that of $\pi\pi$ scattering. However, the statistics is still limited, and this measurement should be repeated when larger samples become available. There is another way to access the S-wave phase without any assumption about its content. This method has been applied to the $K\pi$ system and will be described in the next section.

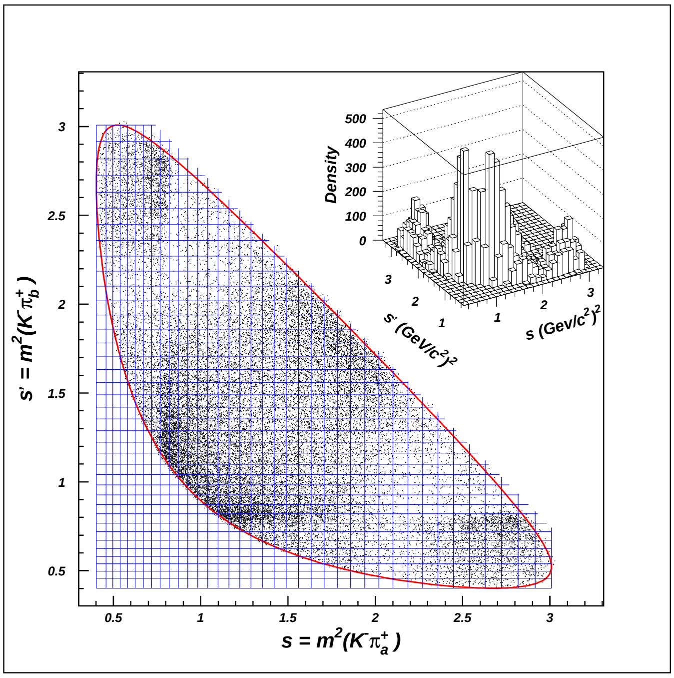

Fig. 1. Dalitz plot for the $D^+ \to K^-\pi^+\pi^+$. The plot is symmetrized, each event appearing twice.

3 The Kπ S-wave amplitude

3.1 The D$^+ \to$ K$^-\pi^+\pi^+$ decay

A model-independent partial-wave analysis (MIPWA) of the $D^+ \to K^-\pi^+\pi^+$ decay was recently performed by E791 [12]. The Dalitz plot is shown in fig. 1. The bands corresponding to the $K^*(982)\pi$ mode is clearly seen. One of the most remarkable features of this Dalitz plot is the asymmetry in each of the $K^*(982)$ bands. This is most readily explained by the interference with the $K\pi$ S-wave component. One can use this interference effect to infer the structure of the S-wave, provided the other components are correctly modeled.

In this work the S-wave amplitude was parameterized by a generic complex function, $\mathcal{A} = f(s)e^{i\phi(s)}$, with s being the $K\pi$ invariant mass squared. The P- and D-waves were parameterized in the framework of the isobar model, as in [2]. The $K\pi$ mass spectrum was divided in bins and, for each bin, the magnitude and the phase of $\mathcal{A}(s_k) = a_k e^{i\phi_k}$ were determined by the fit.

Considering the simplest, tree level diagrams for the $D^+ \to K^-\pi^+\pi^+$ decay, the $K^-\pi^+$ system should be in isospin $I = 1/2$ state. The decay amplitude is written as a sum of $K^-\pi^+$ partial waves labeled by the orbital angular momentum L,

$$A(s_a, s_b) = \sum_L (-2pq)^L P_L(\cos\theta) f_D^L(q, r_D) C_L(s_a), \quad (1)$$

where s_a, s_b are the two independent $K^-\pi^+$ mass squared, $\mathbf{p}$ and $\mathbf{q}$ are the momenta for the K^- and the bachelor pion π_b^+, respectively, in the $K^-\pi_a^+$ rest frame, θ

is the helicity angle, $\cos\theta = \hat{p}\cdot\hat{q}$, and f_D^L is a form factor for the D^+ vertex. The amplitude must be Bose symmetrized with respect to the identical pions π_a^+ and π_b^+.

$$\mathcal{A}(s_a, s_b) = A(s_a, s_b) + A(s_b, s_a). \qquad (2)$$

The main goal is to measure $C_0(s)$,

$$C_0(s) = a_0(s_k)e^{i\phi_0(s_k)} = c_k e^{i\gamma_k}. \qquad (3)$$

The reference waves are

$$C_1(s) = [BW_{K^*(892)}(s) + b_1 BW_{K_1^*(1680)}(s)] f_R^1(p, r_R) \quad (4)$$

and

$$C_2(s) = b_2 BW_{K_2^*(1430)}(s) f_R^2(p, r_R). \qquad (5)$$

A maximum likelihood fit, including representation of the background, effects of the finite detector resolution and correction for the non-uniform acceptance, was performed in order to determine the values of c_k and γ_k. The likelihood function is

$$\mathcal{L} = \prod_{\text{events}} [P_S(M; s_a, s_b) + P_B(M; s_a, s_b)], \qquad (6)$$

where the signal and background probability distribution functions are

$$P_S(M; s_a, s_b) = \frac{g(M)\varepsilon(s_a, s_b)\, |\mathcal{A}(s_a, s_b)|^2}{\int \mathrm{d}s_a \mathrm{d}s_b \mathrm{d}M\, g(M)\varepsilon(s_a, s_b)\, |\mathcal{A}(s_a, s_b)|^2} \qquad (7)$$

and

$$P_B(M; s_a, s_b) = \sum b_k(M) f_k \frac{\mathcal{B}_k(s_a, s_b)}{N_k}, \qquad (8)$$

In the above equations, $g(M)$ and $b_k(M)$ are functions describing the $K^-\pi^+\pi^+$ invariant-mass distribution for the signal and background, and $\varepsilon(s_a, s_b)$ is the acceptance.

The $K\pi$ elastic cross-section measurement is largely dominated by the LASS experiment [13]. LASS measured the $K\pi$ S-wave amplitude from the reaction $K + N \to K\pi + N$. The one-pion exchange is assumed to be the dominant mechanism for this reaction. What LASS actually measured was the scattering amplitude of a real kaon and a virtual pion, off the mass shell, if one neglects other contributions like ρ exchange. The $I = 1/2$ S-wave amplitude, $\mathcal{T} = a(s)e^{i\delta(s)}$, is predominantly elastic up to the $K\eta'$ threshold. If the bachelor pion, π_b^+ in the $D^+ \to K^-\pi_a^+\pi_b^+$ decay acts like a mere spectator, then Watson theorem should hold and the dynamics of the final state would be entirely determined by the $K\pi_a$ system. In this case, the relation between $C_L(s)$ and the corresponding amplitudes $T_L(s)$, measured in scattering experiments, is

$$C_L(s) = \frac{\sqrt{s}}{p} \frac{\mathcal{P}(s) T_L(s)}{p^L f_D^L}, \qquad (9)$$

where $\mathcal{P}(s)$ is an unknown function describing the $K\pi$ production in D decays, with no s-dependent phase. As a consequence, the phases $\phi_L(s)$ and $\delta_L(s)$ should match, up to a constant, for all waves. The validity of the Watson theorem relies on the absence of FSI between $K\pi_a$ and π_b. It has been often assumed to hold, but it has never been tested.

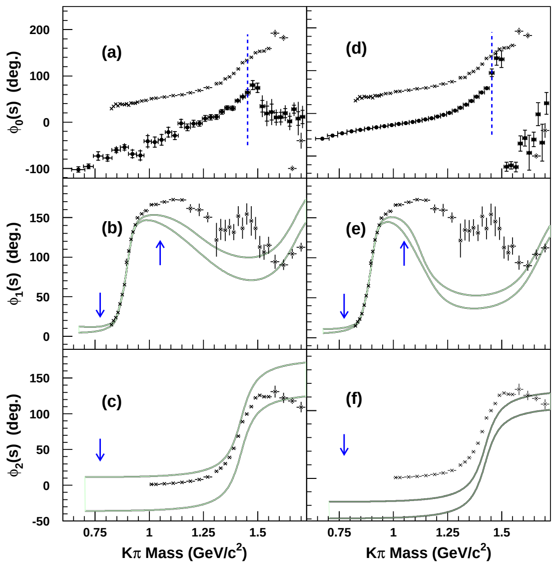

Fig. 2. Results of the best fit to E791 data (panels (a) to (c)), compared to the LASS results (crosses). In (a) the E791 phases are shown as solid circles, whereas in (b) and (c) the solid curves are the E791 P- and D-wave phases (with errors). Panels (d) to (f) show the same quantities from a fit where the S-wave amplitude was fixed to that of the LASS experiment.

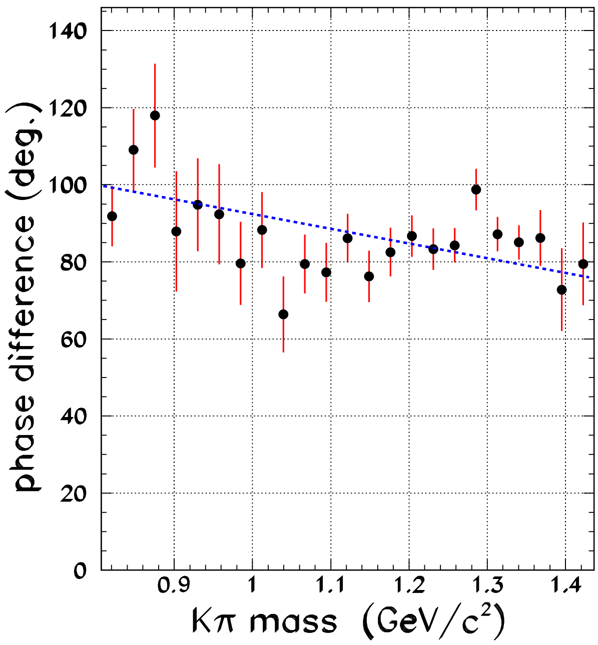

Fig. 3. Difference between S-wave phases from LASS and E791 as a function of the $K\pi$ mass. Error bars are largely dominated by E791 data.

In figs. 2(a)-(c) the result of the MIPWA is shown. In fig. 2(a) the solid circles are the phase $\phi_0(s)$ determined by the MIPWA, whereas in figs. 2(b) and (c) the solid

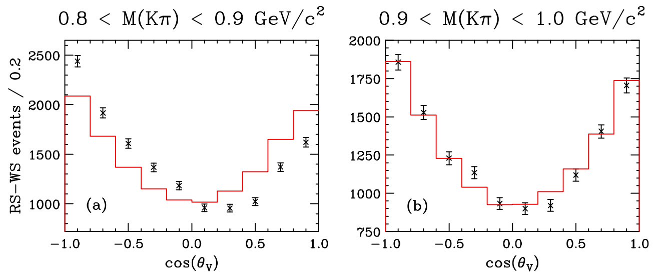

Fig. 4. The $\cos\theta_V$ distribution, split between samples above and below the $0.9\,\mathrm{GeV}/c^2$. Points with error bars are FOCUS data (background subtracted) and solid histogram is a Monte Carlo simulation of the $D^+ \to \bar{K}^*(892)^0\mu^+\nu$ decay.

curves enclose the zones within 1 standard deviation of the MIPWA P- and D-waves. In all plots the crosses are the LASS $I = 1/2$ phases $\delta_L(s)$.

We see that the phases $\phi_0(s)$ and $\delta_0(s)$ are clearly different, not only by an overall shift. They show a different s-dependence, which can be see more clearly in fig. 3, where the two phases have been subtracted. Errors (essentially from E791) are still large, but they will be reduced by a factor of two when the result of the FOCUS experiment MIPWA analysis is released.

A possible origin of the observed discrepancy could be a wrong modeling of the P- and D-waves, which would reflect on the measurement of $\phi_0(s)$. It is interesting to fit the data constraining the shape of the S-wave phase $\phi_0(s)$ to follow that of LASS. The results of such fit are shown in figs. 2(d)-(e). The large offset of the S-wave phase persists, while both P- and D-waves now show even larger differences than before. The phase variation required by Watson theorem is not observed.

3.2 The $D^+ \to K^-\pi^+\mu^+\nu$ decay

Recently, the Fermilab FOCUS Collaboration found a small $K^-\pi^+$ ($\simeq$ 5%) S-wave component in the decay $D^+ \to K^-\pi^+\mu^+\nu$ [14]. The kinematics of this decay is described by five variables. Usually, these variables are the $K^-\pi^+$ and $\mu^+\nu$ invariant mass squared, the angle between the planes defined by the $K^-\pi^+$ and $\mu^+\nu$, the angle between the μ^+ and the D line of flight, in the $\mu^+\nu$ frame, and the angle between the K^- and the D line of flight, in the $K^-\pi^+$ frame, $\cos\theta_V$. The presence of the S-wave is manifested by an asymmetry in the $\cos\theta_V$ distribution, shown in fig. 4. If the $K^-\pi^+$ resulted only from the $K^*(892)$, the $\cos\theta_V$ would be symmetric. A full angular analysis was performed in order to disentangle the S-wave component. It was found that this component was empirically described by a constant amplitude, $\mathcal{A}_0 = 0.36e^{i\pi/2}$.

It is very interesting to study the $K^-\pi^+$ amplitude in semi-leptonic decays. Since in this case there is no strong FSI between the $K^-\pi^+$ and the lepton pair, the Watson theorem should strictly hold. In a second study of this channel [15], the content of the S-wave was investigated. The line shape of the $K^-\pi^+$ peak is rather insensitive to the form of the S-wave, so one has to look

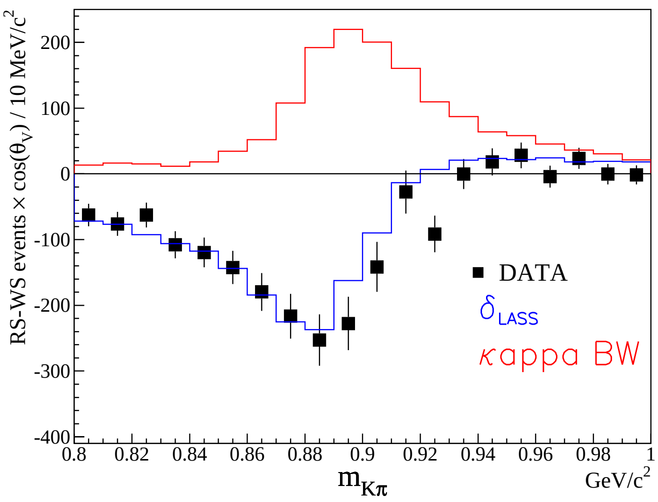

Fig. 5. The background-subtracted $m_{K\pi}$ distribution weighted by $\cos\theta_V$. Data show good agreement with the LASS non-resonant parameterization (solid histogram), but not with the κ.

at the $\cos\theta_V$ distribution in order to distinguish between models. This distribution is well represented if instead of the $\mathcal{A}_0 = 0.36e^{i\pi/2}$ S-wave amplitude one uses the LASS effective-range formula,

$$\mathcal{A}_0 = \frac{p^*}{\sqrt{s}}\,\sin\delta_0(s)e^{i\delta_0(s)}, \qquad (10)$$

with s being the $K^-\pi^+$ mass squared, p^* being the kaon momentum in the $K\pi$ frame and $\delta_0(s)$ given by

$$\cot\delta_0(s) = \frac{1}{ap} + \frac{bp}{2}\,. \qquad (11)$$

This equation represents a non-resonant contribution defined by a scattering length a and an effective range b. A Breit-Wigner representing a possible contribution from the κ-meson would also work, but in this case we should add a $\pi/2$ overall phase.

4 Conclusion

Based on the analysis of high-statistics data on D decays, one can conclude that the $K^-\pi^+$ S-wave phase variation is consistent with the $I = 1/2$ S-wave phase from $K^-\pi^+$ scattering in semi-leptonic decays, but differs substantially in hadronic decays.

There are several possible reasons for that. First, the $K^-\pi^+$ phase extracted by LASS relies on the assumption of a particular reaction mechanism, the one-pion exchange. Other mechanisms could also contribute. In this case the LASS $K^-\pi^+$ phase would not correspond to a pure $K^-\pi^+$ scattering.

In another scenarium, the $K^-\pi^+$ system from the $D^+ \to K^-\pi^+\pi^+$ decay would be a mixture of $I = 1/2$ and $I = 3/2$ states, whereas in the $D^+ \to K^-\pi^+\mu^+\nu$ decay the $K^-\pi^+$ system would be produced only in a

$I = 1/2$ state, assuming the decay mechanism to be the tree level W-radiation amplitude. The $I = 1/2/I = 3/2$ mixture would be different in $D^+ \to K^-\pi^+\pi^+$ decay and $K^-\pi^+$ scattering [16]. Compatibility between LASS and E791 would be achieved comparing the E791 phase to a sum of $I = 1/2$ and $I = 3/2$ phases from LASS. One problem of such hypotesis is that the required amount of the $I = 3/2$ amplitude is large, which implies a large non-resonant component (recall that the $I = 3/2$ amplitude is entirely non-resonant). Such a large non-resonant component would be unique to this particular channel.

The difference between LASS $I = 1/2$ and E791 phases could also be explained by the final-state interactions, present in hadronic final states, but absent in semi-leptonic decays [17]. The effect of the FSI would be to distort the phase variation of the pure $K^-\pi^+$ scattering. Recent calculations [18] show that, once FSI is taken into account, the phases extracted from production and scattering match nicely.

In any case, one cannot extract the κ pole position directly from the S-wave phase measured by the MIPWA.

For the $\pi^-\pi^+$ S-wave amplitude, the method developed by Bediaga and Miranda [11] should be applied for higher statistics samples whenever available. Also, the MIPWA technique used by E791 on the $D^+ \to K^-\pi^+\pi^+$ decay should be performed on the $D^+ \to \pi^-\pi^+\pi^+$, as an alternative method to extract the S-wave phase. In addition, the semi-leptonic decay $D^+ \to \pi^-\pi^+\mu^+\nu$ should be analyzed looking for an S-wave component, in which the phase of the $\pi^-\pi^+$ amplitude could be measured in the absence of FSI. Here, the S-wave $\pi^+\pi^-$ system can be only in $I = 0$ state. If the FSI play no role, the S-wave phase from $D^+ \to \pi^-\pi^+\pi^+$ and from the $I = 0$ CERN-Munich and K_{l4} data should match.

On the theoretical side, the FSI correction to the pure $\pi^-\pi^+/K^-\pi^+$ scattering amplitude should be computed. If this is the origin of the observed discrepancies, then the content of the $\pi^-\pi^+$ and $K^-\pi^+$ S-waves could be extracted from D-meson decays and, in particular, that would allow the determination of the position of the σ and κ poles.

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